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FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF A CIRCULAR ANNULAR PLATE OF RECTANGULAR ORTHOTROPY WITH AN INTERMEDIATE CIRCULAR SUPPORT

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1. INTRODUCTION

Vibrating solid, isotropic circular plates with internal circular or secant supports have been reported in reference [1]. The case of an annular plate with an intermediate circular support and a free inner edge has been recently considered [2]. The present study is an extension of the problem treated in reference [2] to the case of annular plates of rectangular orthotropy, see Figure 1. The fundamental frequency coefficients are determined as a function of the parameters b/a and c/a assuming that the plate is either clamped or simply supported at the outer boundary ($r = a$). The optimized Rayleigh–Ritz method is used to determine the fundamental frequency coefficient and the fundamental mode shape is approximated by means of two polynomial co-ordinate functions which satisfy the essential boundary conditions but do not take into account the azimuthal variation of the mode shape.

This approach has been followed in previous studies [3–4] in the case of solid and circular plates with a free inner edge. Two independent finite element determinations have shown that the accuracy of the results obtained in references [3, 4] is excellent for $b/a \leqslant 0{\cdot}7$ [5]. For larger values of b/a the analytical approach yields frequency values which are very high upper bounds. This is due to the fact that the polynomial co-ordinate functions do not take into account the azimuthal variations of the plate middle surface vibrating in its fundamental mode. In view of this fact it is reasonable to expect that, since the plate

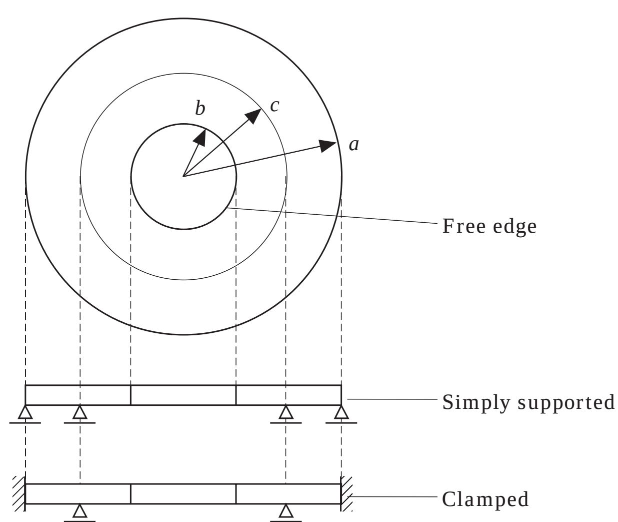

Figure 1. Vibrating annular plate of rectangular orthotropy considered in the present study.

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possesses an intermediate circular support for $r = c$ in the present study, the accuracy of the eigenvalues will be better than the accuracy of those obtained in references [3, 4].

2. APPROXIMATE ANALYTICAL SOLUTION

Following previous studies [3–5] and using Lekhnitskii's well established notation [6] one expresses the governing functional in the form

$$J[W] = \tfrac{1}{2}\int\int\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_1\nu_2\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y - \frac{\rho\omega^2}{2}\int\int hW^2\,\mathrm{d}x\,\mathrm{d}y. \quad (1)$$

The displacement amplitude is now approximated using the truncated expression

$$W(r) \simeq W_a(r) = C_1(\alpha_P r^P + \alpha_Q r^Q + \alpha_2 r^2 + 1) + C_2(\beta_P r^{P+1} + \beta_Q r^{Q+1} + \beta_3 r^3 + 1), \quad (2)$$

where P and Q are Rayleigh's optimization parameters and where the α's and β's are determined by substituting each co-ordinate function in the boundary conditions

$$W(a) = W(c) = 0, \qquad \mathrm{d}W/\mathrm{d}r\,(a) = 0, \quad (3, 4)$$

in the case where the plate is clamped at the outer boundary, or

$$\mathrm{d}^2W/\mathrm{d}r^2 + (\nu_2/r)\,\mathrm{d}W/\mathrm{d}r|_{r=a} = 0, \quad (5)$$

when the plate is simply supported at $r = a$.

Equation (5) is the exact boundary condition in the case of an isotropic circular plate (in this case one has $\nu_2 = \nu$) and for this situation one achieves excellent accuracy using the proposed approach. It is then reasonable to expect that in the case of a circular plate of rectangular orthotropy, where satisfying the natural boundary condition will be an exceedingly difficult task, the use of the approximate condition (5) will yield satisfactory accuracy, at least from a practical viewpoint [3, 5]. Following reference [2] the natural boundary conditions at $r = b$ are not satisfied.

Substituting equation (2) in equation (1) and requiring that

$$\partial J[W]/\partial C_i = 0, \qquad (i = 1, 2), \quad (6)$$

one obtains a homogeneous, linear system of equations in the C_i's.

The non-triviality condition yields a determinantal equation whose lowest root constitutes the fundamental frequency coefficient of the structural system shown in Figure 1, $\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$.

Since the method yields upper bounds one can minimize Ω_1 with respect to P and Q (either independently keeping constant one of them or with respect to both parameters simultaneously). The value of Ω_1 is then optimized.

3. NUMERICAL RESULTS

The numerical determinations of the fundamental frequency coefficient have been performed for a prescribed set of orthotropic parameters ($D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$ and $\nu_2 = 1/3$).

TABLE 1

Values of Ω in the case of circular, annular plates with an outer simply supported boundary. ($D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$; $\nu_2 = 1/3$)

		(A)				(B)			(C)		
b/a	c/a	P	Q	$\Omega_1^{(1)}$	$\Omega_1^{(2)}$	P	Q	$\Omega_1^{(2)}$	P	Q	$\Omega_1^{(2)}$
0·1	0·1	4	3	14·42	13·67	0·1	3	13·06	4	1·1	13·06
0·1	0·2	4	3	16·86	16·68	0·8	3	16·63	4	2·4	16·68
0·1	0·3	4	3	21·27	21·22	1·7	3	21·16	4	7·6	21·06
0·1	0·4	4	3	26·34	26·16	2·2	3	26·12	4	1·4	26·04
0·1	0·5	4	3	25·72	25·58	0·9	3	25·33	4	0·7	25·34
0·1	0·6	4	3	20·38	20·34	1·1	3	20·19	4	0·9	20·19
0·1	0·7	4	3	16·15	16·01	2·1	3	15·97	4	1·4	15·90
0·1	0·8	4	3	13·65	12·99	7·1	3	12·94	4	9·5	12·94
0·1	0·9	4	3	12·34	11·22	15·8	3	10·80	4	17·3	10·82
0·2	0·2	4	3	15·85	15·22	1·9	3	15·09	4	1·8	15·09
0·2	0·3	4	3	20·04	20·00	8·5	3	19·97	4	3·4	20·00
0·2	0·4	4	3	26·21	26·12	2·2	3	26·10	4	1·4	26·06
0·2	0·5	4	3	28·02	27·84	1·1	3	27·68	4	0·6	27·69
0·2	0·6	4	3	22·18	21·91	0·9	3	21·85	4	2·1	21·89
0·2	0·7	4	3	16·94	16·94	1·6	3	16·85	4	1·1	16·79
0·2	0·8	4	3	13·93	13·67	10·5	3	13·57	4	15·5	13·62
0·2	0·9	4	3	12·40	11·78	20·1	3	11·25	4	22·3	11·32
0·3	0·3	4	3	19·15	18·97	4·9	3	18·96	4	3·4	18·96
0·3	0·4	4	3	26·16	26·03	1·4	3	25·98	4	9·3	25·91
0·3	0·5	4	3	33·32	33·12	2·8	3	33·12	4	13·3	33·08
0·3	0·6	4	3	28·23	28·14	9·5	3	28·02	4	5·7	28·00
0·3	0·7	4	3	20·38	20·36	10·7	3	20·31	4	12·5	20·24
0·3	0·8	4	3	15·93	15·57	10·1	3	15·50	4	16·1	15·53
0·3	0·9	4	3	13·78	13·02	21·1	3	12·45	4	24·1	12·54
0·4	0·4	4	3	25·47	25·45	0·1	3	25·36	4	0·1	25·42
0·4	0·5	4	3	38·04	36·35	1·9	3	36·26	4	1·2	36·28
0·4	0·6	4	3	42·40	41·82	0·9	3	41·70	4	10·5	41·69
0·4	0·7	4	3	29·54	29·16	0·7	3	29·02	4	0·1	29·07
0·4	0·8	4	3	21·37	20·20	5·8	3	20·18	4	9·3	20·18
0·4	0·9	4	3	17·67	15·85	18·3	3	15·24	4	20·3	15·28
0·5	0·5	4	3	37·22	36·37	0·1	3	36·11	4	0·1	36·18
0·5	0·6	4	3	61·26	54·71	2·8	3	54·66	4	1·9	54·67
0·5	0·7	4	3	52·69	50·43	0·1	3	49·87	4	0·1	49·99
0·5	0·8	4	3	34·04	30·64	3·1	3	30·63	4	2·3	30·63
0·5	0·9	4	3	26·26	21·68	15·2	3	21·01	4	20·1	21·03
0·6	0·6	4	3	60·19	56·44	0·1	3	56·06	4	0·1	56·13
0·6	0·7	4	3	105·7	89·67	3·6	3	89·67	4	2·7	89·67
0·6	0·8	4	3	66·31	57·80	0·7	3	57·45	4	0·1	57·49
0·6	0·9	4	3	44·99	34·20	12·8	3	33·48	4	12·1	33·46
0·7	0·7	4	3	111·6	99·75	0·1	3	99·28	4	0·1	99·36
0·7	0·8	4	3	177·9	149·9	0·2	3	149·4	4	0·1	149·4
0·7	0·9	4	3	92·93	67·34	9·6	3	66·83	4	8·6	66·80
0·8	0·8	4	3	262·1	223·4	0·1	3	222·8	4	0·1	222·9
0·8	0·9	4	3	286·4	215·6	1·5	3	215·4	4	0·7	215·4
0·9	0·9	4	3	1091·0	891·2	0·1	3	890·4	4	0·1	890·6

Notes: $\Omega_1^{(1)}$ determined with one co-ordinate function; $\Omega_1^{(2)}$ determined with two co-ordinate functions. (A) results obtained without optimization; (B) results obtained optimizing with respect to P; (C) results obtained optimizing with respect to Q.

TABLE 2

Values of Ω_1 in the case of circular, annular plates with an outer clamped boundary ($D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$; $\nu_2 = 1/3$)

		(A)				(B)			(C)		
b/a	c/a	P	Q	$\Omega_1^{(1)}$	$\Omega_1^{(2)}$	P	Q	$\Omega_1^{(2)}$	P	Q	$\Omega_1^{(2)}$
0·1	0·1	4	3	22·54	21·12	1·4	3	20·49	4	1·4	20·48
0·1	0·2	4	3	26·39	25·99	5·7	3	25·94	4	3·7	25·96
0·1	0·3	4	3	32·81	32·81	9·1	3	32·59	4	6·9	32·40
0·1	0·4	4	3	36·21	36·05	0·9	3	35·75	4	0·6	35·82
0·1	0·5	4	3	29·31	28·68	6·3	3	28·55	4	4·2	28·61
0·1	0·6	4	3	21·69	21·68	9·9	3	21·48	4	7·7	21·37
0·1	0·7	4	3	17·09	16·73	3·2	3	16·73	4	12·6	16·59
0·1	0·8	4	3	14·46	13·49	7·9	3	13·35	4	9·3	13·35
0·1	0·9	4	3	12·90	11·61	17·9	3	10·99	4	19·1	11·01
0·2	0·2	4	3	25·12	24·08	3·1	3	24·05	4	2·5	24·04
0·2	0·3	4	3	31·68	31·61	8·2	3	31·44	4	5·3	31·43
0·2	0·4	4	3	38·81	38·81	10·2	3	38·65	4	5·7	38·79
0·2	0·5	4	3	33·60	32·63	4·9	3	32·60	4	3·3	32·62
0·2	0·6	4	3	23·78	23·61	9·1	3	23·43	4	5·6	23·54
0·2	0·7	4	3	17·91	17·83	2·2	3	17·79	4	1·3	17·71
0·2	0·8	4	3	14·75	14·26	10·6	3	14·05	4	14·4	14·11
0·2	0·9	4	3	12·97	12·22	21·3	3	11·47	4	22·9	11·54
0·3	0·3	4	3	30·74	30·50	6·9	3	30·69	4	4·7	30·37
0·3	0·4	4	3	41·78	41·45	0·9	3	41·21	4	9·3	40·93
0·3	0·5	4	3	45·52	45·52	9·2	3	45·31	4	5·9	45·23
0·3	0·6	4	3	31·60	31·55	9·6	3	31·19	4	7·1	31·01
0·3	0·7	4	3	21·91	21·69	2·5	3	21·68	4	12·9	21·47
0·3	0·8	4	3	17·07	16·32	9·9	3	16·16	4	14·6	16·18
0·3	0·9	4	3	14·55	13·56	22·1	3	12·73	4	24·1	12·82
0·4	0·4	4	3	41·21	41·13	0·1	3	40·80	4	8·8	40·69
0·4	0·5	4	3	60·76	57·47	1·3	3	57·03	4	0·5	57·12
0·4	0·6	4	3	52·78	52·37	0·1	3	51·59	4	9·8	51·55
0·4	0·7	4	3	32·90	31·79	1·1	3	31·56	4	12·7	31·50
0·4	0·8	4	3	23·47	21·32	6·7	3	21·24	4	7·6	21·24
0·4	0·9	4	3	18·96	16·60	19·8	3	15·65	4	21·3	15·69
0·5	0·5	4	3	60·40	58·69	0·1	3	57·94	4	0·1	58·20
0·5	0·6	4	3	94·94	84·38	1·6	3	83·93	4	0·7	84·00
0·5	0·7	4	3	61·99	58·27	0·1	3	57·02	4	0·1	57·33
0·5	0·8	4	3	38·46	32·69	4·2	3	32·69	4	3·4	32·69
0·5	0·9	4	3	28·77	22·85	17·3	3	21·71	4	17·4	21·70
0·6	0·6	4	3	97·70	90·60	0·1	3	89·57	4	0·1	89·85
0·6	0·7	4	3	149·5	129·0	0·5	3	128·1	4	0·1	128·1
0·6	0·8	4	3	76·97	63·19	1·4	3	62·86	4	0·7	62·90
0·6	0·9	4	3	50·38	36·34	15·1	3	34·93	4	14·4	34·90
0·7	0·7	4	3	181·1	159·3	0·1	3	157·8	4	0·1	158·2
0·7	0·8	4	3	222·2	185·5	0·1	3	182·8	4	0·1	183·4
0·7	0·9	4	3	106·6	72·16	11·9	3	70·83	4	10·9	70·78
0·8	0·8	4	3	425·9	354·6	0·1	3	352·7	4	0·1	353·1
0·8	0·9	4	3	338·3	236·5	3·1	3	236·4	4	2·2	236·4
0·9	0·9	4	3	1777·0	1405·0	0·1	3	1402·0	4	0·1	1403·0

Notes: $\Omega_1^{(1)}$ determined with one co-ordinate function; $\Omega_1^{(2)}$ determined with two co-ordinate functions. (A) results obtained without optimization; (B) results obtained optimizing with respect to P; (C) results obtained optimizing with respect to Q.

Tables 1 and 2 depict values of Ω_1 for simply supported and clamped out boundariess respectively. Clearly, when $b/a = c/a$ one has the case where the inner boundary is simply supported.

Each table contains a first column (A) where one can evaluate the convergence of the procedure by using a two term approximation versus a single polynomial approximation, for $P = 4$ and $Q = 3$ in equation (2). In other words no minimization of the fundamental eigenvalue has been performed. As b/a increases the effect of using a two term approximation is very marked, as can be appreciated from Tables 1 and 2. The effect is more noticeable in the case of the plate with clamped outer boundary (Table 2).

The tables also allow for the evaluation of:

(1) the effect of optimizing Ω_1 with respect to the exponential parameter P, as Q is kept constant, column (B), when two co-ordinate functions are used.

(2) The effect of optimizing Ω_1 with respect to Q, as P is kept constant, when two co-ordinate functions are used.

It is observed that in some instances (1) is more effective than (2); in others the situation reverses and in some cases they are equivalent. On the other hand: the results contained in column (A) when two co-ordinate functions are used, $\Omega_1^{(2)}$, are always higher than the values depicted in columns (B) and (C).

As c/a approaches unity the value of Ω_1 is practically the same, regarding the type of boundary condition at the outer boundary (see for instance the case where $b/a = 0{\cdot}5$, $c/a = 0{\cdot}9$: $\Omega_1 = 21{\cdot}01$, Table 1 and $\Omega_1 = 21{\cdot}70$, Table 2). Obviously the same situation takes place in the case of isotropic plates [2].

In view of the popular use of orthotropic and, in general, anisotropic materials in all fields of technology, it is hoped that simple approaches as the one presented here will be useful to designers.

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